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Spectral representation of two-point Green superfunction

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Abstract. A supersymmetric version of the Kallen–Lehman procedure is presented for interacting superfields. On the basis of properties of the Fock superspace, the supersymmetric invariance and the asymptotic hypothesis, we show that the exact two-point Green superfunction (TPGF) can be written as a superposition of the corresponding free TPGF with one spectral weight. When the supercurrent is conserved, the spectral weight of vector superfield TPGF is positive and affects only the transverse part of the vector superfield while its longitudinal one remains concentrated at the spurious mass μ^2. We show also that in the case of SQED, where in spite of derivature feature of the interacting Lagrangian, the sum rules of the vector superfield spectral weight are like that of conventional QED.

1. Introduction

It is well known that the Kallen–Lehman procedure [1, 2] gives a spectral representation of the exact two-point Green function. Let us recall that this procedure, based on relativistic invariance arguments and asymptotic hypothesis, allows us to write the exact TPGF as a superposition of the corresponding free TPGF with some spectral weights. In the case of a non-derivative interacting Lagrangian, these weights satisfy sum rules [3].

The purpose of this work is to extend this procedure to the superfields theories to construct spectral representations of exact TPGF of interacting chiral and vector superfields.

This paper is organized as follows: in section 2, we present Dirac's procedure to treat the SQED Lagrangian in terms of chiral and vector superfield components. In section 3, we construct the Fock superspace of particle superstates and present its completeness relation, which exhibits an indefinite metric feature originated from the spurious part of the vector superstates. In section 4, a supersymmetric version of the Kallen–Lehman procedure is applied to vacuum expectation values of the commutators and propagators of interacting chiral and vector superfields. As in conventional field theories, the Kallen–Lehman procedure gives, in superfields theories, a spectral representation of exact TPGF as a superposition of free TPGF with one spectral weight. In spite of the derivative feature of the interacting Lagrangian of SQED, the quantum version of Dirac's brackets gives the same sum rules form of vector superfield spectral weight as in QED. Throughout this paper we use the convention used by Guerdane [4].

2. Dirac's formalism of SQED

The Lagrangian density describing the interaction of a massive vector superfield $V(x, \theta, \theta)$ with two chiral scalar superfields $\Phi_+(x, \theta, \theta)$ and $\Phi_-(x, \theta, \theta)$ reads [3]:

$$L = L_V + L_{\Phi V} + L_{m\Phi} \tag{2.1}$$

where

$$L_V = \frac{1}{8} V D\bar{D}^2 D V + m^2 V^2 - \frac{\varepsilon}{16} V(D^2\bar{D}^2 + \bar{D}^2 D^2) V \tag{2.2a}$$

is the free massive gauge fixing Lagrangian density of $V(x, \theta, \bar{\theta})$ with a mass m (Wess 1983), and D and $\bar{D}$ are the supersymmetric covariant derivatives.

$$L_{\Phi V} = \Phi_+^\dagger \, e^{2eV} \Phi_+ + \Phi_-^\dagger \, e^{-2eV} \Phi_- \tag{2.2b}$$

is the interacting term of SQED, and

$$L_{m\Phi} = -\frac{1}{8} m_\phi \left(\frac{\Phi_- D^2}{\Box \Phi_+} + \frac{\Phi_-^\dagger \bar{D}^2}{\Box \Phi_+^\dagger} \right) = m_\phi [\Phi_+ \Phi_- \delta(\theta) + \Phi_+^\dagger \Phi_-^\dagger \delta(\bar{\theta})] \tag{2.2c}$$

is the mass term of the scalar superfields Φ_+ and Φ_-.

The vector superfield equation of motion reads

$$\frac{1}{8} D\bar{D}^2 D V + m^2 V - \frac{\varepsilon}{16} (D^2\bar{D}^2 + \bar{D}^2 D^2) V = J \tag{2.3}$$

where the supercurrent

$$J = e(\Phi_+^\dagger \, e^{2eV} \Phi_+ - \Phi_-^\dagger \, e^{-2eV} \Phi_-) \tag{2.4}$$

is conserved, i.e. $D^2 J = \bar{D}^2 J = 0$ when the chiral superfields satisfy the equations of motion:

$$\tfrac{1}{4} D^2 (e^{2eV} \Phi_+) = m_\phi \Phi_-^\dagger \qquad \tfrac{1}{4} D^2 (e^{-2eV} \Phi_-) = m_\phi \Phi_+^\dagger \tag{2.5a}$$

$$\tfrac{1}{4} \bar{D}^2 (e^{2eV} \Phi_+^\dagger) = m_\phi \Phi_- \qquad \tfrac{1}{4} \bar{D}^2 (e^{-2eV} \Phi_-^\dagger) = m_\phi \Phi_+ \tag{2.5b}$$

In terms of components, the different parts of (2.1) can be written as

$$L_V = L_{\rm kin} + L_{\rm m} + L_\varepsilon$$

with the kinetic term

$$L_{\rm kin} = \tfrac{1}{2} D^2 + i\partial_n \lambda \sigma^n \bar{\lambda} - \tfrac{1}{4} v^{nm} v_{nm} \tag{2.6a}$$

the mass term

$$L_{\rm m} = m^2 [-\tfrac{1}{2} V^2 - \chi\lambda - \bar{\chi}\bar{\lambda} + \tfrac{1}{2} M^\dagger M + i\partial_n \chi \sigma^n \bar{\chi} + \tfrac{1}{2} C \Box C + CD] \tag{2.6b}$$

and the gauge fixing term

$$L_\varepsilon = -\frac{\varepsilon}{2} \{ M^\dagger \Box M + (D + \Box C)^2 + (\partial V)^2 + 2i\partial_n \lambda \sigma^n \bar{\lambda}$$

$$+ 2i\partial_n \chi \sigma^n \Box \bar{\chi} - 2\lambda \Box \chi - 2\bar{\lambda} \Box \bar{\chi} \} \tag{2.6c}$$

where M, D, C, v_n, λ and χ are, respectively, the scalars, vector and Weyl spinor field composing the vector superfield $V(x, \theta, \bar{\theta})$, $v^{nm} = \partial^n v^m - \partial^m v^n$ and ε is the gauge fixing parameter.

$$\begin{aligned}
L_{V\Phi} = {} & e^{2eC}\left\{-\frac{e}{2}\,\partial_n C\partial^n(A_+^\dagger A_+) - \partial_n A_+^\dagger \partial A_+ + F_+^\dagger F_+ + \frac{i}{2}\,\partial_n\bar{\psi}_+\bar{\sigma}^n\psi_+ - \frac{i}{2}\,\bar{\psi}_+\bar{\sigma}^n\partial_n\psi_+\right\} \\
& - ie\, e^{2eC}\chi\left\{\frac{i}{\sqrt{2}}\,\sigma^n\partial_n\bar{\psi}_+A_+ - \frac{i}{\sqrt{2}}\,\sigma^n\bar{\psi}_+\partial_n A_+ + \sqrt{2}\,\psi_+F_+^\dagger\right\} + \text{h.c.} \\
& + e\, e^{2eC}F_+^\dagger A_+(e\chi^2 + iM) + \text{h.c.} \\
& + e\, e^{2eC}(v^n - e\chi\sigma^n\bar{\chi})[\bar{\psi}_+\bar{\sigma}_n\psi_+ + i(A_+^\dagger\partial_n A_+ - \partial_n A_+^\dagger A_+)] \\
& - \sqrt{2}e\, e^{2eC}A_+^\dagger A_+\left\{D + \tfrac{1}{4}\Box C + e\left[M^\dagger M - v^2 - 2\chi\left(\lambda + \frac{i}{2}\,\sigma^n\partial_n\bar{\chi}\right)\right.\right. \\
& \left.\left. - 2\bar{\chi}\left(\bar{\lambda} + \frac{i}{2}\,\bar{\sigma}^n\partial_n\chi\right)\right] + e^2[i\bar{\chi}^2 M - i\chi^2 M^\dagger + \tfrac{2}{3}\bar{\chi}\bar{\sigma}^n\chi v_n] + e^3\chi^2\bar{\chi}^2\right\} \\
& + (e \to -e \text{ and } + \to - \text{ for the } \Phi_-^\dagger\, e^{-2eV}\Phi_- \text{ terms})
\end{aligned} \tag{2.7}$$

where the fields $A_\pm$, $F_\pm$ and $\psi_\pm$ are the components of the matter superfields $\Phi_\pm$, and

$$L_{m\Phi} = m_\phi[A_+F_- + A_-F_+ - \psi_+\psi_- + \text{h.c.}] \tag{2.8}$$

In terms of components, the Lagrangian density (2.1) presents terms with high-order derivative which require additional independent fields

$$\phi_1 = \dot{\phi} = (C_1 = \dot{C}, \chi_1 = \dot{\chi}, \bar{\chi}_1 = \dot{\bar{\chi}}) \tag{2.9}$$

and their first and second-order conjugate momenta defined by [4]

$$\pi_\phi = \frac{\delta L}{\delta\dot{\phi}} - \frac{d}{dt}\left(\frac{\delta L}{\delta\ddot{\phi}}\right) \qquad \text{and} \qquad \pi_{\phi_1} = \frac{\delta L}{\delta\ddot{\phi}} \tag{2.10}$$

where L is the total Lagrangian and the overdot denotes the time derivative.

The conjugate momenta form of the chiral superfield components give two even primary constraints

$$\theta_{+1} = \pi_{F_+} \simeq 0 \tag{2.11a}$$

$$\theta_{+2} = \pi_{F_+^\dagger} \simeq 0 \tag{2.11b}$$

and four odd primary constraints

$$(\theta_{+5})_\alpha = \left(\pi_{\psi_+} - \frac{i}{2}\,\sigma^0\bar{\psi}_+\, e^{2eC} - \frac{e}{\sqrt{2}}\,\sigma^0\bar{\chi}\, e^{2eC}A_+^\dagger\right)_\alpha \simeq 0 \tag{2.11e}$$

$$(\theta_{+6})^{\dot{\alpha}} = \left(\pi_{\bar{\psi}_+} - \frac{i}{2}\,\bar{\sigma}^0\psi_+\, e^{2eC} + \frac{e}{\sqrt{2}}\,\bar{\sigma}^0\chi\, e^{2eC}A_+\right)^{\dot{\alpha}} \simeq 0. \tag{2.11f}$$

The primary constraints originating from the components of the vector superfield have the same form as the free one [5]:

$$\theta_{V_1}=\pi_D\simeq 0 \tag{2.11g}$$

$$(\theta_{V_3})_\alpha=[\pi_\lambda-\mathrm{i}(1-\varepsilon)\sigma^0\bar{\lambda}]_\alpha\simeq 0 \tag{2.11i}$$

$$(\theta_{V_4})^{\dot\alpha}=\pi^{\dot\alpha}_{\bar\lambda}\simeq 0 \tag{2.11j}$$

$$(\theta_{V_5})_\alpha=(\pi_{\chi_1}+\varepsilon\lambda)_\alpha\simeq 0 \tag{2.11k}$$

$$(\theta_{V_6})^{\dot\alpha}=\pi^{\dot\alpha}_{\bar\chi_1}+\varepsilon(\bar\lambda+\mathrm{i}\bar\sigma\partial\chi+\mathrm{i}\bar\sigma\chi_1)^{\dot\alpha}\simeq 0. \tag{2.11l}$$

Among these constraints, only θ_{+1}, θ_{+2} and θ_{V_1} commute with all the others, and then can give secondary constraints which are simply the equations of motion of the fields F_+, $F^\dagger_+$ and D, respectively:

$$\theta_{+3}=\dot\pi_{F_+}=\mathrm{e}^{2eC}F^\dagger_+ +\mathrm{i}2^{1/2}e\,\mathrm{e}^{2eC}\bar\chi\bar\psi_+ +e\,\mathrm{e}^{2eC}A^\dagger_+(e\bar\chi^2-\mathrm{i}M^\dagger)+m_\phi A_-\simeq 0 \tag{2.11c}$$

$$\theta_{+4}=\dot\pi_{F^\dagger_+}=\mathrm{e}^{2eC}F_+ -\mathrm{i}2^{1/2}e\,\mathrm{e}^{2eC}\chi\psi_+ +e\,\mathrm{e}^{2eC}A_+(e\chi^2+\mathrm{i}M)+m_\phi A^\dagger_-\simeq 0 \tag{2.11d}$$

$$\dot\pi_D=D+m^2C-\varepsilon(D+\Box C)+e\,\mathrm{e}^{2eC}A^\dagger_+A_+ -e\,\mathrm{e}^{-2eC}A^\dagger_-A_-\simeq 0.$$

The form of

$$\pi_{C_1}=-\tfrac{1}{2}mC+\varepsilon(D+\Box C)-\frac{e}{2}\mathrm{e}^{2eC}A^\dagger_+A_+ +\frac{e}{2}\mathrm{e}^{-2eC}A^\dagger_-A_- \tag{2.12}$$

give the following secondary constraint for the massive superfield:

$$\theta_{V_2}=-\pi_{C_1}+D+\tfrac{1}{2}m^2C+\frac{e}{2}\mathrm{e}^{2eC}A^\dagger_+A_+ -\frac{e}{2}\mathrm{e}^{-2eC}A^\dagger_-A_-\simeq 0. \tag{2.11h}$$

The constraints originating from the components of the scalar superfield Φ_- are obtained by replacing the subscripts + by − and the electric charge e by $-e$.

Finally, the theory presents sixteen constraints originating from the components of the chiral superfields Φ_+ and Φ_- and ten constraints originating from the components of the vector superfield V.

Using the Poisson brackets of fundamental canonical variables ϕ, ϕ_1, π_ϕ and π_{ϕ_1} with the following definitions for the fermionic variables

$$\{\phi^\alpha(x,t),\pi_\beta(y,t)\}=-\delta^\alpha_\beta\delta^3(x-y) \tag{2.13a}$$

$$\{\bar\phi_{\dot\alpha}(x,t),\pi^{\dot\beta}(y,t)\}=-\delta^{\dot\beta}_{\dot\alpha}\delta^3(x-y) \tag{2.13b}$$

which are obtained from the left derivative convention used by Wess [5], we observe that the constraints are of the second class and the supermatrix

$$\Omega_{Ai,Bj}(x,y)=\{\theta_{Ai}(x,t),\theta_{Bj}(y,t)\}=-(-)^{ij}\Omega_{Bj,Ai} \tag{2.14}$$

(with $A, B=+, -$ or v and i, j taking the values α, β when the constraints are fermionic) is non-singular.

The non-vanishing elements of $\Omega_{Ai,Bj}$ are:

$$\Omega_{+1,+4}(x,y)=\Omega_{+2,+3}(x,y)=-\mathrm{e}^{2eC(x)}\delta^3(x-y) \tag{2.15a}$$

$$\Omega^{\dot\alpha}_{+3,+6}(x,y)=\mathrm{i}2^{1/2}e\,\mathrm{e}^{2eC(x)}\bar\chi^{\dot\alpha}(x)\delta^3(x-y) \tag{2.15b}$$

$$\Omega_{+4,5\alpha}(x,y)=-\mathrm{i}2^{1/2}e\,\mathrm{e}^{2eC(x)}\chi_\alpha(x)\delta^3(x-y) \tag{2.15c}$$

$$\Omega^{\dot\alpha}_{+5\alpha,+6}(x,y)=-\mathrm{i}\,\mathrm{e}^{2eC(x)}\sigma^{0\dot\alpha}_{\alpha}\delta^3(x-y) \tag{2.15d}$$

$$\Omega^{\dot\alpha}_{+6,+5\alpha}(x,y)=-\mathrm{i}\,\mathrm{e}^{2eC(x)}\bar\sigma^{0\dot\alpha}_{\alpha} \tag{2.15e}$$

$$\Omega_{V_1,V_2}(x,y)=-\delta^3(x-y)\qquad \Omega_{V_3\alpha,V_4\beta}=\varepsilon\varepsilon_{\alpha\beta}\delta^3(x-y) \tag{2.15f}$$

$$\Omega^{\beta}_{V_3\alpha,V_5}(x,y)=\mathrm{i}\,\sigma^{0\beta}_{\alpha}(1-\varepsilon)\qquad \Omega^{\beta}_{V_4\alpha,V_6}(x,y)=\mathrm{i}\varepsilon\sigma^{0\beta}_{\alpha} \tag{2.15g}$$

$$\Omega^{\alpha\beta}_{V_5,V_6}(x,y)=\varepsilon\varepsilon^{\alpha\beta} \tag{2.15h}$$

and $+\to-$, $e\to-e$ for the matrix elements generated from the components of the scalar superfield Φ_-.

These non-vanishing elements show that the supermatrix $\Omega_{Ai,Bj}(x,y)$ is composed of three independent superblocks. Two superblocks originate from the chiral superfield Φ_+ and Φ_- and the third originates from the superfield V. The latter has the same form as that of the free vector superfield [5].

To quantize the theory we must construct the Dirac brackets defined by

$$\{A(x,t),B(y,t)\}^*=\{A(x,t),B(y,t)\}$$
$$-\int \mathrm{d}^3x'\,\mathrm{d}^3x''\,\{A(x,t),\theta_{Ai}(x',t)\}\Omega^{Ai,Bj}(x',x'')\{\theta_{Bj}(x'',t),B(y,t)\} \tag{2.16}$$

where $\Omega^{Ai,Bj}$ is the inverse matrix of $\Omega_{Ai,Bj}$ whose non-vanishing elements are

$$\Omega^{+1,+4}=\Omega^{+2,+3}=-\Omega^{+3,+2}=-\Omega^{+4,+1}=\mathrm{e}^{-2eC(x)}\delta^3(x-y) \tag{2.17a}$$

$$\Omega^{+1,+2}=-\Omega^{+2,+1}=2\mathrm{i}e\,\mathrm{e}^{-2eC(x)}\chi\sigma^0\chi \tag{2.17b}$$

$$\Omega^{+1,+6}_{\dot\alpha}=\Omega^{+6,+1}_{\dot\alpha}=2^{1/2}e\,\mathrm{e}^{-2eC(x)}\bar\sigma^{0\alpha}_{\dot\alpha}\chi_\alpha \tag{2.17c}$$

$$\Omega^{+5\alpha,+2}=\Omega^{+2,5\alpha}=-2^{1/2}e\,\mathrm{e}^{-2eC(x)}\sigma^{0\alpha}_{\dot\alpha}\bar\chi^{\dot\alpha} \tag{2.17d}$$

$$\Omega^{+5\alpha,+6}_{\dot\alpha}=-\mathrm{i}\,\mathrm{e}^{-2eC(x)}\sigma^{0\alpha}_{\dot\alpha} \tag{2.17e}$$

$$\Omega^{+6,+5\alpha}_{\dot\alpha}=-\mathrm{i}\,\mathrm{e}^{-2eC(x)}\bar\sigma^{0\alpha}_{\dot\alpha} \tag{2.17f}$$

$$\Omega^{V_1,V_2}=-\Omega^{V_2,V_1}=\delta^3(x-y) \tag{2.17g}$$

$$\Omega^{V_3\alpha,V_4\beta}=-\varepsilon^{\alpha\beta}\delta^3(x-y) \tag{2.17h}$$

$$\Omega^{V_3\dot\alpha,V_5}_{\beta}=\Omega^{V_5,V_3\dot\alpha}_{\beta}=-\mathrm{i}\bar\sigma^{0\dot\alpha}_{\beta} \tag{2.17i}$$

3. The Fock superspace

The spectral representation of two-point superfunctions requires the construction of the Fock superspace. This construction needs the canonical quantization of the free supersymmetric field theory based on Dirac's brackets (2.16) where the non-vanishing supermatrix elements $\Omega^{Ai,Bj}(x,y)$ are given by (2.17) with a null electric charge e.

The superfield equations of motion of the free fields $V(x,\theta,\bar\theta)$, $\Phi_+(x,\theta,\bar\theta)$ and $\Phi_-(x,\theta,\bar\theta)$ are given by (2.3), (2.5*a*) and (2.5*b*), respectively, with $e=0$.

$$\tfrac{1}{8}D\bar D^2DV+m^2V-\tfrac{1}{16}\varepsilon\,(D^2\bar D^2+\bar D^2D^2)V=0 \tag{3.1}$$

$$\tfrac{1}{4}\Phi_\pm(x,\theta,\bar\theta)=m_\phi^2\Phi^\dagger_\mp(x,\theta,\bar\theta) \tag{3.2}$$

$$\tfrac{1}{4}\Phi^\dagger_\pm(x,\theta,\bar\theta)=m_\phi^2\Phi_\mp(x,\theta,\bar\theta) \tag{3.3}$$

Acting D^2 and $\bar{D}^2$ on equation (3.1), we obtain:

$$(\Box-\mu^2)D^2V=0 \qquad \text{and} \qquad (\Box-\mu^2)\bar{D}^2V=0 \tag{3.4}$$

which show that the superfields D^2V and $\bar{D}^2V$ are free and have the same mass $\mu^2 = m^2/\varepsilon$. Equation (2.3) shows that they remain free even if the superfield V is coupled to a current J satisfying the conservation equations:

$$D^2J=\bar{D}^2J=0 \tag{3.5}$$

The canonical quantization of the massive vector multiplet was studied in [4] where it was shown that the vector superfield V can be written as

$$V(x,\theta,\bar{\theta})=\int \widetilde{\mathrm{d}k}_m\, V_m(k,\theta,\bar{\theta})\,\mathrm{e}^{ikx}+\int \widetilde{\mathrm{d}k}_\mu\, V_\mu(k,\theta,\bar{\theta})\,\mathrm{e}^{ikx}+\text{h.c.} \tag{3.6}$$

where $\widetilde{\mathrm{d}k}_{m(\mu)}=\mathrm{d}^3k(2\pi)^3 2\omega_{m(\mu)}$ and $\omega_{m(\mu)}=k^0=(k^2+m^2(\mu^2))^{1/2}$ and

$$\begin{aligned}V_m(k,\theta,\theta)={}&-\mathrm{d}(k)/m^2+\mathrm{i}\theta k\bar{b}(k)/m^2-\mathrm{i}\bar{\theta}\bar{k}a(k)/m^2-\theta v(k)\bar{\theta}+\mathrm{i}\theta^2\bar{\theta}\bar{b}(k)/2\\&-\mathrm{i}\bar{\theta}^2\theta a(k)/2+\theta^2\bar{\theta}^2\,\mathrm{d}(k)/4\end{aligned} \tag{3.7}$$

is the physical super-annihilation operator of square mass m^2, containing annihilation operators of the different physical components of the superfield V, and

$$\begin{aligned}V_\mu(k,\theta,\bar{\theta})={}&c_\mu(k)+\mathrm{i}\theta e(k)-\mathrm{i}\bar{\theta}\bar{f}(k)+\mathrm{i}\theta^2(m(k)-\mathrm{i}n(k))/2-\mathrm{i}\bar{\theta}^2(m(k)-\mathrm{i}n(k))/2\\&-\theta k\bar{\theta}v_s+\mathrm{i}\theta^2\bar{\theta}\bar{k}e(k)/2-\mathrm{i}\bar{\theta}^2\theta kf(k)+\theta^2\bar{\theta}^2\mu^2c_\mu/4\end{aligned} \tag{3.8}$$

is the spurious super-annihilation operator of square mass μ^2 containing the annihilation operators of the different spurious components of the superfield V.

These superoperators satisfy the following commutation relations

$$\begin{aligned}&[V_m(k,\theta,\bar{\theta}),V_m^+(k',\theta',\bar{\theta}')]\\&\quad=\frac{1}{4m^2}\exp(\theta k\bar{\theta}'-\theta' k\bar{\theta})[4-m^2\delta^4(\theta-\theta')](2\pi)^3 2\omega_m\delta^3(k-k')\end{aligned} \tag{3.9}$$

with $k^2=-m^2$, and

$$\begin{aligned}&[V_\mu(k,\theta,\bar{\theta}),V_\mu^+(k',\theta',\bar{\theta}')]\\&\quad=-\frac{1}{4m^2}\exp(\theta k\bar{\theta}'-\theta' k\bar{\theta})[4+\mu^2\delta^4(\theta-\theta')](2\pi)^3 2\omega_\mu\delta^3(k-k')\end{aligned} \tag{3.10}$$

with $k^2=-\mu^2$.

From (3.6), (3.9) and (3.10), we obtain the commutator of the two vector superfields as

$$\begin{aligned}&[V(x,\theta,\bar{\theta}),V(x',\theta',\bar{\theta}')]\\&\quad=\frac{1}{4\Box}\exp[\mathrm{i}(\theta'\partial\bar{\theta}-\theta\partial\bar{\theta}')]((4-\Box\delta^4(\theta-\theta'))\mathrm{i}\Delta(x-x',m^2)\\&\qquad-(4+\Box\delta^4(\theta-\theta'))\mathrm{i}\Delta(x-x',\mu^2)/\varepsilon)\end{aligned} \tag{3.11}$$

where

$$i\Delta(x-x', m^2(\mu^2)) = \int \mathrm{d}\tilde{k}_m[e^{ik(x-x')} - e^{-ik(x-x')}] \qquad \text{with } k^2 = -m^2(\mu^2)$$

and the vacuum expectation value of the time product of two free massive vector superfields as

$$\langle 0|TV(x,\theta,\bar{\theta}), V(x',\theta',\bar{\theta}')|0\rangle$$
$$= -\frac{1}{4\Box}\exp[i(\theta'\not{\partial}\bar{\theta} - \theta\not{\partial}\bar{\theta}')]((4-\Box\delta^4(\theta-\theta'))G_f(x-x', m^2)$$
$$-(4+\Box\delta^4(\theta-\theta'))G_f(x-x', \mu^2)/\varepsilon) \tag{3.12}$$

where $G_f(x-x', m^2(\mu^2))$ is the Feynman scalar propagator of square mass $m^2(\mu^2)$.

We follow the same procedure to quantize the matter superfields Φ_+ and Φ_-. Acting D^2 on the equations (3.2) we obtain the following equations of motion:

$$(\Box - m_\phi^2)\Phi_\pm(x,\theta,\bar{\theta}) = 0 \tag{3.13}$$

whose solutions are:

$$\Phi_\pm(x,\theta,\bar{\theta}) = \int \mathrm{d}\tilde{k}[A_\pm(k,\theta,\bar{\theta})\, e^{ikx} + B_\pm^\dagger(k,\theta,\bar{\theta})\, e^{-ikx}] \tag{3.14}$$

with $\mathrm{d}\tilde{k} = \mathrm{d}^3k/(2\pi)^3 2\omega$ and $\omega = k^0 = (k^2 + m_\phi^2)^{1/2}$.

The conditions $\bar{D}\Phi_\pm(x,\theta,\bar{\theta}) = D\Phi_\pm^\dagger(x,\theta,\bar{\theta}) = 0$ give solutions of the form:

$$\begin{aligned} A_\pm(k,\theta,\bar{\theta}) &= e^{-\theta k\bar{\theta}}A_\pm(k,\theta) \qquad & A_\pm^\dagger(k,\theta,\bar{\theta}) &= e^{-\theta k\bar{\theta}}A_\pm^\dagger(k,\bar{\theta}) \\ B_\pm(k,\theta,\bar{\theta}) &= e^{\theta k\bar{\theta}}B_\pm(k,\bar{\theta}) \qquad & B_\pm^\dagger(k,\theta,\bar{\theta}) &= e^{\theta k\bar{\theta}}B_\pm^\dagger(k,\theta) \end{aligned} \tag{3.15}$$

with the following expansions:

$$A_\pm(k,\theta) = a_\pm(k) + (2)^{1/2}\theta d_\pm(k) + \theta^2 f_\pm(k) \tag{3.16a}$$

$$B_\pm(k,\bar{\theta}) = b_\pm(k) + (2)^{1/2}\bar{\theta}\bar{c}_\pm(k) + \bar{\theta}^2 g_\pm(k) \tag{3.16b}$$

where $(a_\pm(k), b_\pm^\dagger(k))$, $(d_{\pm\alpha}(k), c_{\pm\alpha}(k))$ and $(f_\pm(k), g_\pm^\dagger(k))$ are, respectively, the Fourier expansion of the superfield components $A_\pm(x)$, $\psi_\pm(x)$ and $F_\pm(x)$.

The equations of motion (3.2) and (3.3) and the canonical quantization of their components give the following commutation relations:

$$[A_\pm(k,\theta), A_\pm^\dagger(k',\bar{\theta}')] = (2\pi)^3 2k^0\delta(k-k')\exp(2\theta k\bar{\theta}') \tag{3.17a}$$

$$[B_\pm(k,\bar{\theta}), B_\pm^\dagger(k',\theta')] = (2\pi)^3 2k^0\delta(k-k')\exp(-2\theta' k\bar{\theta}) \tag{3.17b}$$

$$[A_\pm(k,\theta), B_\mp^\dagger(k',\theta')] = -m_\phi(2\pi)^3 2k^0\delta(k-k')(\theta-\theta')^2 \tag{3.17c}$$

$$[B_\pm(k,\bar{\theta}), A_\mp^\dagger(k',\bar{\theta}')] = -m_\phi(2\pi)^3 2k^0\delta(k-k')(\bar{\theta}-\bar{\theta}')^2 \tag{3.17d}$$

which lead to the free superfields commutators

$$[\Phi_\pm(x,\theta,\bar{\theta}), \Phi_\pm^\dagger(x',\theta',\bar{\theta}')] = i\, e^{i(\theta\not{\partial}\bar{\theta} + \theta'\not{\partial}\bar{\theta}' - 2\theta\not{\partial}\bar{\theta}')}\Delta(x-x', m_\phi^2) \tag{3.18a}$$

$$[\Phi_\pm(x,\theta,\bar{\theta}), \Phi_\mp(x',\theta',\bar{\theta}')] = -m_\phi(\theta-\theta')^2\, e^{i(\theta\not{\partial}\bar{\theta} - \theta'\not{\partial}\bar{\theta}')}\Delta(x-x', m_\phi^2). \tag{3.18b}$$

In order to construct the Fock superspace of the superstates, we define the vacuum state $|0\rangle$ of unital norm, which is invariant under the supersymmetric transformations:

$$P_\nu|0\rangle=0 \qquad Q_\alpha|0\rangle=0 \qquad \bar{Q}_{\dot\alpha}|0\rangle=0 \tag{3.19}$$

and annihilated by the action of the annihilation superoperator

$$\begin{aligned} &V_m(k,\theta,\bar\theta)|0\rangle=0 \qquad V_\mu(k,\theta,\bar\theta)|0\rangle=0 \\ &A_\pm(k,\theta,\bar\theta)|0\rangle=B_\pm(k,\theta,\bar\theta)|0\rangle=0. \end{aligned} \tag{3.20}$$

The action of the creation superoperators V_m^+, $A_\pm^\dagger$ and $B_\pm^\dagger$ on the vacuum state define physical multiplet superstates (one-superparticule state),

$$\begin{aligned} &|1_{\rm p}\rangle=V_m^+(k,\theta,\bar\theta)|0\rangle \\ &|A_\pm\rangle=A_\pm^\dagger(k,\theta,\bar\theta)|0\rangle \quad \text{and} \quad |B_\pm\rangle=B_\pm^\dagger(k,\theta,\bar\theta)|0\rangle \end{aligned} \tag{3.21}$$

and $V_\mu^\dagger$ gives the spurious multiplet superstate

$$|1_{\rm s}\rangle=V_\mu^+(k,\theta,\bar\theta)|0\rangle. \tag{3.22}$$

The most general superstate is a mixture of a physical and a spurious multiplet

$$|N\rangle=|N_{\rm p}\rangle\otimes|N_{\rm s}\rangle\otimes|N_{A_+}\rangle\otimes|N_{A_-}\rangle\otimes|N_{B_+}\rangle\otimes|N_{B_-}\rangle \tag{3.23}$$

where $|N_i\rangle$ is a N_i-multiplet superstate constructed from the N_i tensorial product of the corresponding one-particle multiplet superstates.

The chiral superfield equations of motion in the momentum space read

$$\bar{D}^2A_\pm^\dagger(k,\theta,\bar\theta)=4m_\phi B_\mp^\dagger(k,\theta,\bar\theta) \qquad D^2B_\pm^\dagger(k,\theta,\bar\theta)=4m_\phi A_\mp^\dagger(k,\theta,\bar\theta) \tag{3.24a}$$

$$\bar{D}^2B_\pm(k,\theta,\bar\theta)=4m_\phi A_\mp(k,\theta,\bar\theta) \qquad D^2A_\pm(k,\theta,\bar\theta)=4m_\phi B_\mp(k,\theta,\bar\theta) \tag{3.24b}$$

and the commutation relations (3.17) show that either $|N_{A_\pm}\rangle$ or $|N_{B_\pm}\rangle$ are enough to construct the completeness relation of the chiral Fock subspace

$$\begin{aligned} |N_{B_\pm}\rangle\langle N_{B_\pm}| &= \sum_{N=1}^{\infty}\sum_{n+m=N}\int\prod_{i=1}^{n}\frac{{\rm d}^8z_i}{m_\phi^2}\prod_{j=1}^{m}\frac{{\rm d}^8z_j}{m_\phi^2}B_+^\dagger(z_i)B_-^\dagger(z_j)|0\rangle\langle 0|B_+(z_i)B_-(z_j) \\ &= \sum_{N=1}^{\infty}\sum_{n+m=N}\int\prod_{i=1}^{n}\frac{{\rm d}^8z_i}{m_\phi^2}\prod_{j=1}^{m}\frac{{\rm d}^8z_j}{m_\phi^2}\frac{D_i^2\bar{D}_i^2}{16m_\phi^2} \\ &\quad\times A_-^\dagger(z_i)\frac{D_j^2\bar{D}_j^2}{16m_\phi^2}A_+^\dagger(z_j)|0\rangle\langle 0|A_-(z_i)A_+(z_j) \\ &= |N_{A_\pm}\rangle\langle N_{A_\pm}|=1 \end{aligned} \tag{3.25}$$

where we have used ${\rm d}z_i=({\rm d}\tilde{k}_i,{\rm d}\theta_i,{\rm d}\bar\theta_i)$ and

$$\frac{D_i^2\bar{D}_i^2}{16m_\phi^2}A_\pm^\dagger(z_i)=A_\pm^\dagger(z_i). \tag{3.26}$$

Therefore, the full completeness relation reads

$$\begin{aligned} |N\rangle\langle N| = \sum_{N=1}^{\infty}\sum_{(n+N_++N_-=N)}\int\prod_{i=1}^{{\rm p+s}}{\rm d}^8z_i\prod_{i=1}^{N_+}\prod_{j=1}^{N_-}\frac{{\rm d}^8z_i\,{\rm d}^8z_j}{m_\phi^4}\frac{(-2)^{{\rm p+s}}(\varepsilon)^{\rm s}}{s!p!} \\ \times|N_{\rm p},N_{\rm s},N_{A_+},N_{A_-}\rangle\langle N_{\rm p},N_{\rm s},N_{A_+},N_{A_-}| \end{aligned} \tag{3.27}$$

where $n = \mathrm{p} + \mathrm{s}$ and ε is the gauge fixing parameter. This completeness relation shows the indefinite metric feature of the Fock superspace.

4. Spectral representation of exact superfield TPGF

The construction of spectral representations of exact superfield TGPF needs. In addition to the completeness relation of Fock superspace, the supersymmetric invariance and the asymptotic hypothesis. The asymptotic used here is in the same spirit as that in conventional field theories. Intuitively, if we look at any system long enough before the collision has taken place, then the system will be described by particle superstates generated by free superfields $V^{\mathrm{in}}(x, \theta, \bar{\theta})$ and $\Phi^{\mathrm{in}}(x, \theta, \bar{\theta})$. The asymptotic hypothesis states:

$$\Phi_{\pm} \xrightarrow[\lim t \to -\infty]{} Z_{\pm}^{1/2} \Phi_{\pm}^{\mathrm{in}} \tag{4.1a}$$

$$V \xrightarrow[\lim t \to -\infty]{} (Z_3)^{1/2} V_{\mathrm{p}}^{\mathrm{in}} + (Z_3 z)^{1/2} V_{\mathrm{s}}^{\mathrm{in}} \tag{4.1b}$$

where $\Phi_{\pm}^{\mathrm{in}}$ are the free chiral superfields and $V_{\mathrm{p}}^{\mathrm{in}}$ and $V_{\mathrm{s}}^{\mathrm{in}}$ are respectively the physical and spurious parts of the free vector superfield V^{in} which are irreducible under the supersymmetric transformations. The normalization factors Z_3 and $Z_3 z$ are *a priori* different because the dynamic affects differently the physical and the spurious parts of the vector superfield.

If we assume that the one-particle multiplet superstate $|1\rangle$ is stable, the asymptotic assumption states:

$$\langle 0|\Phi_{\pm}|1\rangle = (Z_{\pm})^{1/2} \langle 0|\Phi_{\pm}^{\mathrm{in}}|1\rangle \tag{4.2a}$$

$$\langle 0|V|1\rangle = (Z_3)^{1/2} \langle 0|V_{\mathrm{p}}^{\mathrm{in}}|1\rangle + (Z_3 z)^{1/2} \langle 0|V_{\mathrm{s}}^{\mathrm{in}}|1\rangle. \tag{4.2b}$$

4.1. Vector superfields

Let us consider now the vacuum expectation value of the commutator of interacting vector superfields:

$$\Delta_{\mathrm{vv}} = \langle 0|[V(x, \theta, \bar{\theta}), V(x', \theta', \bar{\theta}')]|0\rangle. \tag{4.3}$$

From the supersymmetric invariance of the theory, we can write

$$\begin{aligned}\Delta_{\mathrm{vv}} &= \langle V|L^{-1}(x, \theta, \bar{\theta}) \cdot L(x', \theta', \bar{\theta}')|V\rangle - (x \leftrightarrow x', \theta \leftrightarrow \theta', \bar{\theta} \leftrightarrow \bar{\theta}') \\ &= \langle V| \exp\{P^{\nu}[\mathrm{i}(x-x')_{\nu} + \theta\sigma_{\nu}\bar{\theta}' - \theta'\sigma_{\nu}\bar{\theta}]\} \cdot V(0, \theta' - \theta, \bar{\theta}' - \bar{\theta})|0\rangle \\ &\quad - (x \leftrightarrow x', \theta \leftrightarrow \theta', \bar{\theta} \leftrightarrow \bar{\theta}')\end{aligned} \tag{4.4}$$

where $|V\rangle = V(0, 0, 0)|0\rangle$ and $L(x, \theta, \bar{\theta}) = \exp[-\mathrm{i}xP + \mathrm{i}\theta Q + \mathrm{i}\bar{\theta}\bar{Q}]$.

Inserting the completeness relation (3.27) of the free asymptotic superstates and the identity $\int \mathrm{d}^4 q\, \delta(k_n - q) = 1$, we obtain:

$$\begin{aligned}\Delta_{\mathrm{vv}} = \int \mathrm{d}^4 q \sum_N \delta^4(q - k_N) \langle V|N\rangle\langle N|V(0, \theta' - \theta, \bar{\theta}' - \bar{\theta})|0\rangle\, \mathrm{e}^{[\mathrm{i}q(x-x') + \theta q\bar{\theta}' + \theta' q\bar{\theta}]} \\ - (x \leftrightarrow x', \theta \leftrightarrow \theta', \bar{\theta} \leftrightarrow \bar{\theta}').\end{aligned} \tag{4.5}$$

Now, we define the superdensity as

$$\rho(q, \theta'-\theta, \bar{\theta}'-\bar{\theta}) = (2\pi)^3 \sum_N \delta^4(q-k_N)\langle V|N\rangle\langle N|V(0, \theta'-\theta, \bar{\theta}'-\bar{\theta})|0\rangle \tag{4.6}$$

which vanishes when q is not in the forward light cone owing to the condition on the spectrum of the impulsion:

$$q^\nu q_\nu \leqslant 0 \qquad q^0 \geqslant 0.$$

Applied to V and V', the equations (3.5) give

$$\begin{aligned}
&\left[-\left(\frac{\partial^2}{\partial\bar{\theta}\,\partial\bar{\theta}}\right)+2(\theta-\theta')\not{q}\left(\frac{\partial}{\partial\theta}\right)+q^2(\theta-\theta')^2\right](q^2+\mu^2)\rho(q, \theta'-\theta, \bar{\theta}'-\bar{\theta})=0\\
&\left[-\left(\frac{\partial^2}{\partial\theta\,\partial\theta}\right)+2(\bar{\theta}-\bar{\theta}')\not{q}\left(\frac{\partial}{\partial\theta}\right)+q^2(\theta-\theta')^2\right](q^2+\mu^2)\rho(q, \theta'-\theta, \bar{\theta}'-\bar{\theta})=0\\
&\left[-\left(\frac{\partial^2}{\partial\bar{\theta}'\,\partial\bar{\theta}'}\right)-2(\theta'-\theta)\not{q}\frac{\partial}{\partial\bar{\theta}'}+q^2(\theta'-\theta)^2\right](q^2+\mu^2)\rho(q, \theta'-\theta, \bar{\theta}'-\bar{\theta})=0\\
&\left[-\left(\frac{\partial^2}{\partial\theta'\,\partial\theta'}\right)-2(\bar{\theta}'-\bar{\theta})\bar{\not{q}}\frac{\partial}{\partial\theta'}+q^2(\theta'-\theta)^2\right](q^2+\mu^2)\rho(q, \theta'-\theta, \bar{\theta}'-\bar{\theta})=0.
\end{aligned} \tag{4.7}$$

The function $\rho(q, \theta, \bar{\theta})$ can be written as a general Taylor expansion in θ and $\bar{\theta}$:

$$\begin{aligned}
\rho(q, \theta, \bar{\theta}) = {} & \Gamma(q)+\theta F(q)+\bar{\theta}\bar{F}'(q)+\theta^2 G(q)+\bar{\theta}^2 G'(q)\\
& -\theta\sigma^\nu\bar{\theta}T_\nu(q)+\theta^2\bar{\theta}\bar{\Lambda}(q)+\bar{\theta}^2\theta\Lambda'(q)+\theta^2\bar{\theta}^2\Sigma(q)
\end{aligned} \tag{4.8}$$

where the coefficients: $\Gamma, F, \ldots, \Sigma$ are functions of q. By identification in (4.6) we can write:

$$\begin{aligned}
\Gamma(q) &= (2\pi)^3 \sum_N \delta^4(q-k_N)\langle C|N\rangle\langle N|C\rangle\\
F_\alpha(q) &= (2\pi)^3 \sum_N \delta(q-k_N)\mathrm{i}\langle C|N\rangle\langle N|\chi_\alpha\rangle\\
\bar{F}'_{\dot{\alpha}}(q) &= -(2\pi)^3 \sum_N \delta^4(q-k_N)\mathrm{i}\langle C|N\rangle\langle N|\bar{\chi}_{\dot{\alpha}}\rangle\\
G(q) &= \frac{\mathrm{i}}{2}(2\pi)^3 \sum_N \delta^4(q-k_N)\langle C|N\rangle\langle N|M+\mathrm{i}N\rangle\\
G'(q) &= -\frac{\mathrm{i}}{2}(2\pi)^3 \sum_N \delta^4(q-k_N)\langle C|N\rangle\langle N|M-\mathrm{i}N\rangle\\
T^\nu(q) &= (2\pi)^3 \sum_N \delta^4(q-k_N)\langle C|N\rangle\langle N|A^\nu\rangle\\
\bar{\Lambda}^{\dot{\alpha}}(q) &= \mathrm{i}(2\pi)^3 \sum_N \delta^4(q-k_N)\langle C|N\rangle\left\langle N|\lambda+\frac{\mathrm{i}}{2}\partial\chi\right\rangle\\
\Lambda'_\alpha(q) &= -\mathrm{i}(2\pi)^3 \sum_N \delta^4(q-k_N)\langle C|N\rangle\left\langle N|\lambda+\frac{\mathrm{i}}{2}\partial\chi\right\rangle
\end{aligned} \tag{4.9}$$

$$\Sigma = \frac{(2\pi)^3}{2} \sum_N \delta^4(q-k_N)\langle C|N\rangle\langle N|D+\tfrac{1}{2}\Box C\rangle.$$

In these equations the fields C, D, M, N, A_ν, λ, χ and their derivatives are taken at $x=0$.

The equations (4.7) show that the functions $F(q)$, $\bar{F}'(q)$, $G(q)$, $G'(q)$, $T_\nu(q)$, $\bar{\Lambda}(q)$, $\Lambda'(q)$ and $2\Sigma - \frac{1}{2}q^2\Gamma(q)$ are concentrated at $q^2 = -\mu^2$. They result from the spurious one-multiplet superstates contribution in Δ_{vv}. Consequently, the sum in their expressions is over the spurious one-multiplet only. The completeness relation reduces then to the projector

$$P_s = -2\varepsilon \int \widetilde{\mathrm{d}k}\; d^4\theta |1_s\rangle\langle 1_s| \tag{4.10}$$

with $k^2 = -\mu^2$. Using the expressions of V_μ^{in} and $V_\mu^{\mathrm{in}^+}$ in terms of components creation and annihilation operators, and after an integration over θ and $\bar{\theta}$, P_s becomes:

$$\begin{aligned} P_s = -\int \widetilde{\mathrm{d}k}\; [&m^2 c_\mu^+(k)|0\rangle\langle 0|c_\mu(k) + m^2 v_s^+(k)|0\rangle\langle 0|v_s(k) \\ &+ \varepsilon m^2 \bar{e}(k) k|0\rangle\langle 0|e(k) + \varepsilon m^2 f(k) k|0\rangle\langle 0|f(k) \\ &+ \varepsilon m^+(k)|0\rangle\langle 0|m(k) + \varepsilon n^+(k)|0\rangle\langle 0|n(k)]. \end{aligned} \tag{4.11}$$

This relation with the expressions given by the equations (4.9), and the asymptotic condition (4.2) show that the densities F, F', G, G', Λ, Λ' and T^ν, vanish. Let us consider, for instance, the density T^ν. Inserting the projector P_s we obtain

$$T^\nu(q) = (2\pi)^3 \int \widetilde{\mathrm{d}k}\; \mathrm{d}^4\theta\; \delta(q-k)\langle C|1_s\rangle\langle 1_s|A^\nu\rangle \tag{4.12}$$

with

$$\begin{aligned} \langle C|1_s\rangle &= -(Z_3)^{1/2}\langle D^{\mathrm{in}}/m^2|1_s\rangle + (Z_3 z)^{1/2}\langle (D+\Box C)^{\mathrm{in}}/\mu^2|1_s\rangle \\ \langle 1_s|A^\nu\rangle &= (Z_3)^{1/2}\langle 1_s|(A^{\mathrm{in}\nu} - (1/\mu^2)\partial^\nu\,\partial A^{\mathrm{in}})\rangle + (Z_3 z)^{1/2}\langle 1_s|\partial^\nu\,\partial A^{\mathrm{in}}/\mu^2\rangle \end{aligned} \tag{4.13}$$

From the Fourier expansion of D^{in}, C^{in} and $A^{\mathrm{in}\nu}$ [4], and the commutation relations of the different operator components we show easily that T^ν vanishes. Similarly, we can deduce the vanishing of: G, G', Λ, Λ', F and F'. Now, from the Lorentz invariance and the equations (4.7), we can see that $\Gamma(q)$ and $\Sigma(q)$ are scalar functions of q^2 and satisfy the relation

$$(q^2+\mu^2)(2\Sigma(q^2) - \tfrac{1}{2}q^2\Gamma(q^2)) = 0 \tag{4.14}$$

which means that $2\Sigma(q^2) - \frac{1}{2}q^2\Gamma(q^2)$ differ at most by a multiplicative term from $\delta(q^2+\mu^2)$. For convenience we take

$$2\Sigma - \tfrac{1}{2}q^2\Gamma = -(Z_3 z/\varepsilon)\delta(q^2+\mu^2) \tag{4.15}$$

and we introduce the spectral density

$$\sigma(q^2) = -2\Sigma(q^2) - \tfrac{1}{2}q^2\Gamma(q^2) \tag{4.16}$$

and hence we obtain

$$\Gamma = -\sigma/q^2 + (Z_3 z/q^2\varepsilon)\delta(q^2+\mu^2) \tag{4.17}$$

$$\Sigma = -\tfrac{1}{4}(\sigma + (Z_3 z/\varepsilon)\delta(q^2+\mu^2)). \tag{4.18}$$

Then, Δ_{vv} may be written:

$$\Delta_{vv} = \exp[-i(\theta\not\partial\bar{\theta}' + \theta'\not\partial\bar{\theta})] \int \frac{d^4q}{(2\pi)^3} [\Gamma(q^2) + (\theta-\theta')^4\Sigma(q^2)][e^{iq(x-x')} - e^{-iq(x-x')}] \tag{4.19}$$

and by introducing the spectral density σ, we can write

$$\Delta_{vv} = -\frac{i}{4\Box}\exp[-i\theta\not\partial\bar{\theta}' + i\theta'\not\partial\bar{\theta}] \left\{ [\Box(\theta-\theta')^4 - 4]\left[\int_0^\infty dM^2\,\sigma(M^2)\Delta(M^2, x-x')\right] + (Z_3 z/\varepsilon)[\Box(\theta-\theta')^4 + 4]\Delta(\mu^2, x-x')\right\}. \tag{4.20}$$

To obtain this expression, we have used the identities

$$\int dM^2\,\delta(q^2+\mu^2) = 1 \qquad \int \frac{d^3q}{2q^0} = \int d^4q\,\delta(q^2+\mu^2)\Theta(q^0) \tag{4.21}$$

with $\Theta(q^0) = 1$ if $q^0 \geqslant 0$, and null if $q^0 < 0$.

The exact TGPF thus exhibits a physical part which is a superposition of free commutators with a weight σ, and a non-physical part concentrated at the mass μ^2 and proportional to the spurious free commutator.

The stability of the one-multiplet superstates (4.2) allows us to evaluate the contribution to Δ_{vv} of the one-multiplet superstate.

The contribution of the physical one-multiplet superstates is

$$-2\int d^3k_1\,d^4\theta_1\,\langle 0|V(x,\theta,\bar{\theta})V_m^{in^+}(k_1,\theta_1,\bar{\theta}_1)|0\rangle\langle 0|V_m^{in}(k_1,\theta_1,\bar{\theta}_1)V(x,\theta,\bar{\theta})|0\rangle - (x\leftrightarrow x', \theta\leftrightarrow\theta', \bar{\theta}\leftrightarrow\bar{\theta}'). \tag{4.22}$$

By (4.2*b*), this expression becomes

$$-2Z_3\int d^3k_1\,d^4\theta_1\,\langle 0|V_p^{in}(x,\theta,\bar{\theta})V_m^{in^+}(k_1,\theta_1,\bar{\theta}_1)|0\rangle \times \langle 0|V_m^{in}(k_1,\theta_1,\bar{\theta}_1)V_p^{in}(x',\theta',\bar{\theta}')|0\rangle - [(x,\theta,\bar{\theta})\leftrightarrow(x',\theta',\bar{\theta}')]. \tag{4.23}$$

Using the Fourier expansion (3.6) for V_p^{in} and the commutation relations (3.9) we arrive at the result:

$$-\frac{iZ_3}{4\Box}\exp[i(\theta'\not\partial\bar{\theta} - \theta\not\partial\bar{\theta}')]\cdot[-4+\Box(\theta-\theta')^4]\Delta(m^2, x-x'). \tag{4.24}$$

The contribution of the spurious one-multiplet superstates is evaluated in the same way to give:

$$-\frac{\mathrm{i}Z_3 z}{4\varepsilon\Box}\exp[\mathrm{i}(\theta'\not{\partial}\bar{\theta}-\theta\not{\partial}\bar{\theta}')]\cdot[4+\Box(\theta-\theta')^4]\Delta(\mu_1^2 x-x'). \tag{4.25}$$

The term (4.24) is the contribution to Δ_{vv} concentrated at m^2:

$$\sigma(m^2)=Z_3\delta(M^2-m^2). \tag{4.26}$$

On the other hand, we conclude that the Z_3z-term in Δ_{vv} results from the contribution of the spurious one-multiplet superstates.

If we denote by m_1 the threshold of the more than one-multiplet superstates, we can separate the discrete one-multiplet contribution from the continuum:

$$\begin{aligned}\Delta_{vv}=-\frac{\mathrm{i}}{4\Box}\exp(-\mathrm{i}\theta\not{\partial}\bar{\theta}'+\mathrm{i}\theta'\not{\partial}\bar{\theta})\Big\{&[\Box(\theta-\theta')^4-4]Z_3\cdot\Delta(m^2,x-x')\\&+[\Box(\theta-\theta')^4-4]\int_{m_1^2}^{\infty}\mathrm{d}M^2\,\sigma(M^2)\Delta(M^2,x-x')\\&+\frac{Z_3z}{\varepsilon}[\Box(\theta-\theta')^4+4]\Delta(\mu^2,x-x')\Big\}.\end{aligned} \tag{4.27}$$

The exact propagator $\langle 0|TV(x,\theta,\bar{\theta}),V(x',\theta',\bar{\theta})|0\rangle$ can be obtained from (4.27) by simply replacing $\Delta(x-x',M^2)$ by the Feynman scalar propagator $G_f(x-x',M^2)$ of square mass M^2.

Following the same strategy as in conventional electromagnetic field theory [2], we can obtain, from the superequation of motion (2.3) and the superfield TPGF (4.27), the vacuum expectation value of the commutator of supercurrents:

$$\begin{aligned}&\langle 0|[J(x,\theta,\theta),J(x',\theta',\theta')]|0\rangle\\&\quad=\frac{\mathrm{i}}{4\Box}\exp[\mathrm{i}(\theta'\not{\partial}\bar{\theta}-\theta\not{\partial}\bar{\theta}')][4-\Box\delta^4(\theta-\theta')]\\&\qquad\times\int_{m_1^2}^{\infty}\mathrm{d}M^2\,\sigma(M^2)(m^2-M^2)\Delta(x-x',M^2)\end{aligned} \tag{4.28}$$

which exhibits the disappearance of one superstate contribution, $\langle 0|J|1\rangle=0$ and current conservation. The form of the vector superfield TPGF (4.27) shows also

$$\langle 0|[J(x,\theta,\bar{\theta}),\bar{D}'^2V(x',\theta',\bar{\theta}')]|0\rangle=\langle 0|[J(x,\theta,\bar{\theta}),D'^2V(x',\theta',\bar{\theta}')]|0\rangle=0 \tag{4.29}$$

which can readily be extended to

$$[J(x,\theta,\bar{\theta}),\bar{D}'^2V(x',\theta',\bar{\theta}')]=[J(x,\theta,\bar{\theta}),D'^2V(x',\theta',\bar{\theta}')]=0 \tag{4.30}$$

which reminds us that D^2V and $\bar{D}'^2V$ are free vector superfields (spurious vector superfields), and $\langle 0|J(x,\theta,\bar{\theta})|N\rangle=0$ when $|N\rangle$ includes spurious superstates. This shows that the contribution of the completeness relation in (4.28) does not include intermediate spurious states which involve minus signs, showing that all terms in

$$\sum_N \langle 0|J(x, \theta, \bar{\theta})|N\rangle\langle N|J(x', \theta', \bar{\theta}')|0\rangle$$

contribute with a positive sign; then $\sigma(M^2)$ is positive.

In order to get the sum rules of the weight function $\sigma(M^2)$, let us detail the canonical commutation relations of the interacting vector superfield components. Generally, the interacting Lagrangian depends on the vector superfield $V(x, \theta, \bar{\theta})$ whose components include derivations. Then, in terms of components, we are in the presence of a derivative interacting Lagrangian which can change the form of the conjugate momenta as well as the superconstraint matrix. In this case the canonical commutation relations differ from the free case. However, if the interaction source is external (the supercurrent J does not depend on V) or, in the case of SQED, where, in spite of the derivative feature of the interacting Lagrangian, the superconstraint matrix and Dirac's brackets of the interacting vector superfield components keep the same form as if they were free.

From (4.27), we can deduce the exact TPGF of different vector superfield components. For the vector field A^ν, we find

$$\langle 0|[A_\alpha(x), A_\beta(x')]|0\rangle = \frac{iZ_3}{\mu^2}\,\partial_\alpha\partial_\beta\Delta(x-x', \mu^2)$$
$$+\,i(g_{\alpha\beta}+\partial_\alpha\partial_\beta/\Box)\int_0^\infty dM^2\,\sigma(M^2)\Delta(x-x', M^2) \qquad (4.31)$$

which is exactly the same as the expression obtained from conventional field theory [3].

For these fields, the interacting Lagrangian (2.7) is not derivative, then we get the same sum rules

$$\int_0^\infty dM^2\,\sigma(M^2) = 1 \qquad (4.32)$$

$$Z_3 z = \int_0^\infty dM^2\,\frac{\sigma(M^2)^{M^2}}{M^2} \qquad (4.33)$$

modulo a modification of the normalization of the canonical commutation relations [3]:

$$[A_\nu(x, t), \pi^\mu(x', t)] = i\delta^3(x-x')(g^\mu_\nu - a g^0_\nu g^{\mu 0}) \qquad (4.34a)$$

where

$$a = Z_3 z - 1 = \int_0^\infty dM^2\,\sigma(M^2)\,\frac{(m^2-M^2)}{M^2}. \qquad (4.35)$$

In spite of the derivative interacting Lagrangian (2.7) of the other vector superfield components, Dirac's brackets (2.16) and the equation (4.27) give, for instance,

$$\langle 0|\{\chi_\alpha(x, t), \bar{\chi}_\beta(x', t)\}|0\rangle = 0$$
$$= -\sigma^0_{\alpha\beta}\delta^3(x-x')\left[-\left(\frac{1}{m^2}\right)Z_3 z + \int_0^\infty dM^2\,\frac{\rho(M^2)}{M^2}\right] \qquad (4.36)$$

which give exactly the sum rule (4.33). The sum rules (4.32) and (4.33) remain unchanged for the components of (4.27) if we consider the following canonical commutation relations:

$$[M(x,t),\pi_M(x',t)]=[M^*(x,t),\pi^*_{M^*}(x',t)]=\mathrm{i}\delta^3(x-x')(1+a) \quad (4.34b)$$

$$[C_1(x,t),D(x',t)]=\mathrm{i}\delta^3(x-x') \qquad [C_1(x,t),\pi_{C_1}(x',t)]=\mathrm{i}\delta^3(x-x')(1+a) \quad (4.34c)$$

$$\{\lambda^\alpha(x,t),\pi_{\lambda\beta}(x',t)\}=\mathrm{i}\delta^\alpha_\beta(\varepsilon-1)\delta^3(x-x') \quad (4.34d)$$

$$\{\bar\chi_{1\dot\alpha}(x,t),\pi^{\dot\beta}_{\bar\chi_1}(x',t)\}=-\mathrm{i}\delta^{\dot\beta}_{\dot\alpha}\delta^3(x-x')(1+a) \quad (4.34e)$$

$$\{\bar\lambda_{\dot\alpha}(x,t),\bar\chi_{1\dot\beta}(x',t)\}=\mathrm{i}\varepsilon_{\dot\alpha\dot\beta}\delta^3(x-x'). \quad (4.34f)$$

Note this change in the canonical commutation relations affects only the spurious part of the vector superfield components.

Concerning the Dirac field

$$\psi_D=\begin{pmatrix}\lambda\\ m\chi\end{pmatrix}$$

the TPGF yields one spectral function:

$$\langle 0|\{\psi_D(x),\bar\psi_D(x')\}|0\rangle$$
$$=\begin{pmatrix}\mathrm{i}m & \partial\!\!\!/\\ \bar{\partial\!\!\!/}m^2/\Box & \mathrm{i}m\end{pmatrix}\int_0^\infty \mathrm{d}M^2\,\sigma(M^2)\Delta(x-x',M^2)$$
$$-\mathrm{i}Z_3z\begin{pmatrix}0&0\\ \bar{\partial\!\!\!/}&0\end{pmatrix}\Delta(x-x',\mu^2) \quad (4.37)$$

while conventional theory gives two spectral functions.

4.2. *Chiral superfield*

To construct the spectral representation of the extact TPGF of the interacting chiral superfield, we follow the same strategy as in the vector superfield case. We can express the vacuum expectation value of the commutator of two chiral superfields as

$$\Delta_{\Phi_\pm\Phi_\pm^\dagger}=\langle 0|[\Phi_\pm(x,\theta,\bar\theta),\Phi^\dagger_\pm(x',\theta',\bar\theta')]|0\rangle$$
$$=\langle\Phi_\pm|\,\mathrm{e}^{\mathrm{i}p[(x-x')+\theta\sigma\bar\theta'-\theta'\sigma\bar\theta]}\Phi^\dagger_\pm(0,\theta'-\theta,\bar\theta'-\bar\theta)|0\rangle$$
$$-\langle\Phi^\dagger_\pm|\,\mathrm{e}^{-\mathrm{i}p[(x-x')+\theta'\sigma\bar\theta-\theta\sigma\bar\theta']}\Phi_\pm(0,\theta'-\theta,\bar\theta'-\bar\theta)|0\rangle$$
$$=\int \mathrm{d}^4q\sum_N\delta^4(q-k_N)\langle\Phi_\pm|N\rangle\langle N|\Phi^\dagger_\pm(0,\theta'-\theta,\bar\theta'-\bar\theta)|0\rangle$$
$$\times\,\mathrm{e}^{\mathrm{i}q(x-x')}+\theta q\!\!\!/\bar\theta'-\theta' q\!\!\!/\bar\theta$$
$$-\int \mathrm{d}^4q\sum_N\delta^4(q-k_N)\langle\Phi^\dagger_\pm|N\rangle\langle N|\Phi_\pm(0,\theta'-\theta,\bar\theta'-\bar\theta)|0\rangle$$
$$\times\,\mathrm{e}^{-\mathrm{i}q(x-x')+\theta' q\!\!\!/\bar\theta-\theta q\!\!\!/\bar\theta'}$$
$$=\int\frac{\mathrm{d}^4q}{(2\pi)^3}\{\rho_{\pm\pm1}(q,\theta'-\theta,\bar\theta'-\bar\theta)\,\mathrm{e}^{\mathrm{i}q(x-x')+\theta q\!\!\!/\bar\theta'-\theta' q\!\!\!/\bar\theta}$$
$$-\rho_{\pm\pm2}(q,\theta'-\theta,\bar\theta'-\bar\theta)\,\mathrm{e}^{-\mathrm{i}q(x-x')+\theta' q\!\!\!/\bar\theta-\theta q\!\!\!/\bar\theta'}\} \quad (4.38)$$

where $\langle\Phi_\pm| = \langle 0|\Phi_\pm(0,0,0)$ and $\langle\Phi_\pm^\dagger| = \langle 0|\Phi_\pm^\dagger(0,0,0)$, and

$$\rho_{\pm\pm1}(q, \theta'-\theta, \bar\theta'-\bar\theta) = (2\pi)^3 \sum_N \delta^4(q-k_N)\langle\Phi_\pm|N\rangle\langle N|\Phi_\pm^\dagger(0, \theta'-\theta, \bar\theta'-\bar\theta)\rangle$$

$$\rho_{\pm\pm2}(q, \theta'-\theta, \bar\theta'-\bar\theta) = (2\pi)^3 \sum_N \delta^4(q-k_N)\langle\Phi_\pm^\dagger|N\rangle\langle N|\Phi_\pm(0, \theta'-\theta, \bar\theta'-\bar\theta)\rangle$$

are the superdensities.

The chiral conditions $\bar D\Phi_\pm = 0$ and $D\Phi_\pm^\dagger = 0$, and Lorentz invariance give solutions of superdensities as

$$\rho_{\pm\pm1} = e^{-(\theta'-\theta)\not q(\bar\theta'-\bar\theta)}\rho_{\pm\pm}(q^2)$$
$$\rho_{\pm\pm2} = e^{(\theta'-\theta)\not q(\bar\theta'-\bar\theta)}\rho_{\pm\pm}(q^2)$$

which gives the spectral representation of the interacting chiral superfield TPGF as a superposition of free commutator contribution with a weight $\rho_{\pm\pm}$

$$\Delta_{\Phi_\pm\Phi_\pm^\dagger} = \mathrm{i}\, e^{i(\theta\not\partial\bar\theta + \theta'\not\partial\bar\theta' - 2\theta\not\partial\bar\theta')} \int_0^\infty dM^2\, \rho_{\pm\pm}(M^2)\Delta(x-x', M^2). \tag{4.39}$$

The same procedure gives the vacuum expectation value of the commutator $[\Phi_+, \Phi_-]$ as

$$\begin{aligned}\Delta_{\Phi_+\Phi_-} &= \langle 0|[\Phi_+(x,\theta,\bar\theta), \Phi_-(x',\theta',\bar\theta')]|0\rangle \\ &= \int \frac{d^4q}{(2\pi)^3}\{\rho_{+-1}(q, \theta'-\theta, \bar\theta'-\bar\theta)\, e^{iq(x-x')+\theta\not q\bar\theta'-\theta'\not q\bar\theta} \\ &\quad -\rho_{+-2}(q, \theta'-\theta, \bar\theta'-\bar\theta)\, e^{-iq(x-x')+\theta'\not q\bar\theta-\theta\not q\bar\theta'}\}\end{aligned} \tag{4.40}$$

where

$$\rho_{+-1} = (2\pi)^3 \sum_N \delta^4(q-k_N)\langle\Phi_+|N\rangle\langle N|\Phi_-(0, \theta'-\theta, \bar\theta'-\bar\theta)\rangle$$
$$\rho_{+-2} = (2\pi)^3 \sum_N \delta^4(q-k_N)\langle\Phi_-|N\rangle\langle N|\Phi_+(0, \theta'-\theta, \bar\theta'-\bar\theta)\rangle.$$

The condition $D\Phi_+ = 0$ and $D\Phi_- = 0$ gives solutions of the form

$$\rho_{+-1} = (\theta'-\theta)^2\, e^{(\theta'-\theta)\not q(\bar\theta'-\bar\theta)}\rho_{+-}(q)$$
$$\rho_{+-2} = (\theta'=\theta)^2\, e^{(\theta'-\theta)\not q(\bar\theta'-\bar\theta)}\rho_{+-}(q)$$

and the Lorentz invariance leads to

$$\Delta_{\Phi_+\Phi_-} = \mathrm{i}(\theta-\theta')^2\, e^{i(\theta\not\partial\bar\theta-\theta'\not\partial\bar\theta')} \int_0^\infty dM^2\, \rho_{+-}(M^2)\Delta(x-x', M^2). \tag{4.41}$$

The contribution of the one-multiplet superstate and the asymptotic condition (4.2*a*) give the following interacting chiral TPGF:

$$\begin{aligned}\Delta_{\Phi_\pm\Phi_\pm^\dagger} &= \mathrm{i}Z_\pm\, e^{i(\theta\not\partial\bar\theta + \theta'\not\partial\bar\theta' - 2\theta\not\partial\bar\theta')}\Delta(x-x', m_\phi^2) \\ &\quad + \mathrm{i}\, e^{i(\theta\not\partial\bar\theta + \theta'\not\partial\bar\theta' - 2\theta\not\partial\bar\theta')} \int_{m_{1_\phi}^2}^\infty dM^2\, \rho_{\pm\pm}(M^2)\Delta(x-x', M^2)\end{aligned} \tag{4.42}$$

$$\Delta_{\Phi_+\Phi_-} = im_\phi(Z_+Z_-)^{1/2}(\theta-\theta')^2\, e^{i(\theta\partial\!\!\!/\bar\theta - \theta'\partial\!\!\!/\bar\theta')}\Delta(x-x', m_\phi^2)$$
$$+ i(\theta-\theta')^2\, e^{i(\theta\partial\!\!\!/\bar\theta - \theta'\partial\!\!\!/\bar\theta')}\int_{m_{1_\phi}^2}^{\infty} dM^2\, \rho_{+-}(M^2)\Delta(x-x', M^2) \tag{4.43}$$

where $m_{1_\phi}^2 > m_\phi^2$ is the multiparticle threshold.

As for the vector superfield the chiral superfield TPGF can be written as a superposition of the corresponding free TPGF.

In terms of chiral superfield components equation (4.42) gives

$$\langle 0|\{\psi_{\pm\alpha}(x,t), \bar\psi_{\pm\dot\alpha}(x',t)|0\rangle$$
$$= \sigma^0_{\alpha\dot\alpha}Z_\pm\delta^3(x-x') + \sigma^0_{\alpha\dot\alpha}\delta^3(x-x')\int_{m_{1_\phi}^2}^{\infty} dM^2\, \rho_{\pm\pm}(M^2). \tag{4.44}$$

In section 2, it is shown that the derivative feature of the interacting Lagrangian does not change the constraint matrix relating to the vector superfield components, but changes that of the chiral superfield components. Then the quantum version of Dirac's bracket $\{\psi_{\pm\alpha}(x,t), \bar\psi_{\pm\dot\alpha}(x',t)\}^*$ is different from that of the free fields. Therefore, we cannot obtain for the chiral superfield the same sum rule (4.32) and (4.33).

5. Conclusion

The construction of the Fock superspace based on a covariant canonical quantization of the vector superfield allows us to apply a supersymmetric version of the Kallen–Lehman procedure. In the frame of this procedure we obtained a spectral representation of TPGF of interacting chiral and vector superfields as a superposition of the corresponding free TPGF with one real and positive spectral weight for the vector superfields. The canonical quantization based on Dirac's brackets shows that, in spite of the derivative feature of the interacting Lagrangian of SQED, we obtain the same sum rules as in non-derivative conventional field theories. Let us recall that in conventional vector field theories, the sum rules (4.32) and (4.33) can hold only if we modify the normalization of the canonical commutation relation of the spurious part of the vector field [3]. In vector superfield theories, the same sum rules can hold if the supersymmetric partners of the spurious part of the vector field satisfy modified normalization of the canonical commutation relations.

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